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THE INITIAL SECTION OF A PLANE NONISOTHERMAL JET DEVELOPING UNDER ASYMMETRIC CONDITIONS WITH AND WITHOUT COMBUSTION

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A solution of the problem obtained with the method of integral relations is given here. It is obtained in the form of simple algebraic formulas for the boundaries of the mixing zones on both sides of the jet, the position of the flame front, and the axis of the jet at the end of the initial section. It is found that asymmetry of the boundary conditions has a substantial effect on the characteristics of the jet: the position of the axis of the jet at the end of the initial section can change its sign, depending on the temperature ratio at the boundaries of the jet and for given ratios of the velocity at the boundaries of the jet to the velocity of the jet outflow, the length of the initial section can change several fold.

Development of a gasdynamic theory and perfection of the methods for calculation of the primary zone in the combustion chamber as well as working out of recommendations for updating prospective schemes of gas-turbine plants of different types require solution of some gasdynamic problems on the flow in the primary zone with and without combustion. One such problem is a plane nonisothermal jet developing in asymmetric conditions with and without combustion.

At present there are only works on the theory of combustion of an axisymmetric flame with the same conditions at its boundary, which does not correspond to the flow pattern in the primary zone of the combustion chamber, where the fuel gas is supplied from the nozzle as a radial jet that is transformed subsequently into a circular jet. Between this jet and the wall there is a circular air flow from the swirler, and in the center a weak circulating flow of combustion products appears due to ejection of a transverse jet in the downstream flow. Thus, the circular jet of fuel develops under asymmetric conditions and the flame front exists only on one side, on the side of the air flow. Generally speaking, if not all of the oxygen was used in combustion, it can get into the reverse flow and a second flame front can arise on the side of the flow of combustion products. In any case combustion occurs in substantially asymmetric conditions.

With the radius of the chamber R_c exceeding substantially the thickness of the circular jet, we can assume that $R_c \to \infty$ and consider not a circular but a plane jet with different conditions on its upper (on the side of the oxidant) and the lower (on the side of circulating combustion products) boundaries.

Now we will consider a solution of the problem on the initial plane nonisothermal jet developing in asymmetric conditions with and without combustion with the method of integral relations.

1. Let the jet flow out of an infinitely long slot of width b_0 with velocity u_0 and temperature T_0 into a space in which from the one side a flow moves with velocity $u_{\delta 1}$ and temperature $T_{\delta 1}$ and from the other, a flow moves with the velocity $u_{\delta 2}$ and temperature $T_{\delta 2}$. Since the conditions at the boundaries of the jet are different, mixing zones are developed in different conditions and at the end of the initial section the boundaries of the core converge to a point not on the line $y = 0$ emerging from the center of the slot but on some other line that should be determined in the process of solution, i.e., in this case the axis of the jet does not coincide with the line $y_0(x)$. A schematic diagram of the initial section of the jet developing in asymmetric conditions is shown in Fig. 1a.

It is assumed that in the initial section the velocity and temperature profiles are similar and described by known expressions, in this case for description of the flow it is necessary to determine five quantities, namely, y_{11},

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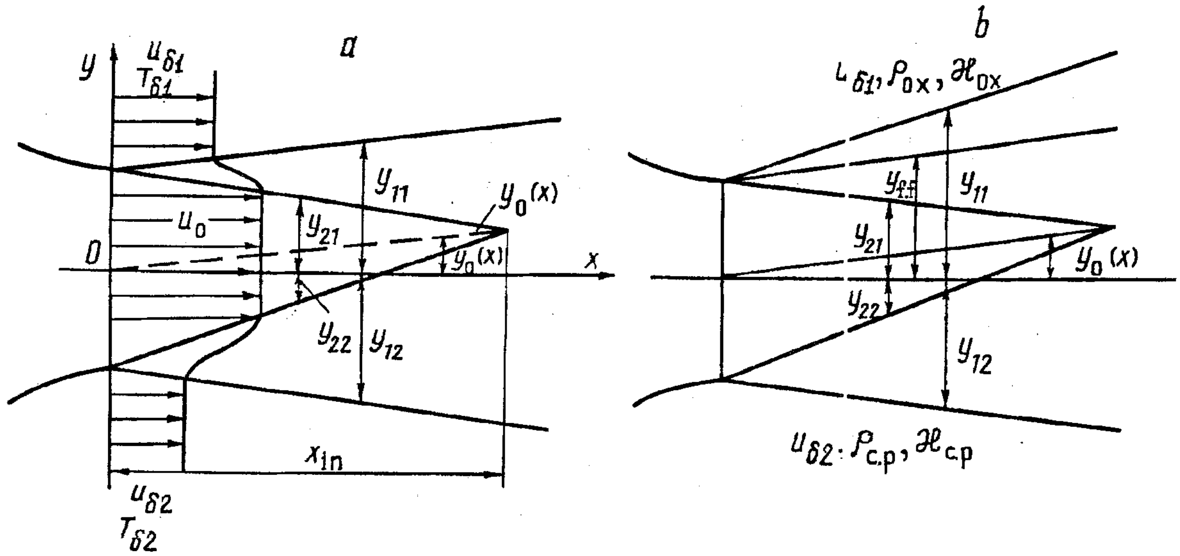


Fig. 1. Schemic diagram of a jet with asymmetric boundary conditions: a) nonisothermal jet; b) jet with a combustion flame.

y_{21}, y_{12}, y_{22}, and y_0. In accordance with the method of integral relations one can use two integral conditions of conservation of excess momentum of the jet in two parts of the jet developing in different conditions and two respective integral energy equations. For determination of y_0 one can use the condition that the boundaries of the potential core in the upper and lower zones converge at a point. Since the pressure across the jet may be assumed constant, the velocity in the potential core of the jet is also invariable, and it follows from the continuity equation that the transverse velocity is zero over the whole region of the potential core. Integral relations for determination of four boundaries of the mixing zones have the form [1]:

$$\frac{d}{dx}\int\limits_0^{y_{1i}(x)} \rho u\,(u - u_{\delta i})\,dy = 0\,, \qquad\qquad (1.1)$$

$$\frac{d}{dx}\int\limits_{y_0}^{y_{1i}} \rho u\,(u - u_{\delta i})^2\,dy = 2\int\limits_{y_0}^{y_{1i}} \overline{\rho u' v'}\,\frac{\partial\,(u - u_{\delta i})}{\partial y}\,dy$$

($i = 1$ for the upper part of the jet and $i = 2$ for its lower part).

The boundary conditions were used in derivation of the integral relations:

$$u = u_{\delta i}\,, \quad \frac{\partial u}{\partial y} = 0 \quad \text{when} \quad y = y_{1i}\,; \qquad\qquad (1.2)$$

$$u = u_0\,, \quad \frac{\partial u}{\partial y} = 0 \quad \text{when} \quad y = y_{2i}\,.$$

It will be assumed that in the cross-sections of the mixing zones the velocity profiles are similar and described by a third-power polynomial, whose coefficients are determined from boundary conditions (1.2), i.e.,

$$\frac{u - u_{\delta i}}{u_0 - u_{\delta i}} = f\,(\eta_i) = 1 - 3\eta_i^2 + 2\eta_i^3\,, \qquad\qquad (1.3)$$

where

$$\eta_i = \frac{y - y_{2i}}{\delta_i}\,. \qquad\qquad (1.4)$$

After some transformations, substitution of Eqs. (1.3) and (1.4) in Eq. (1.1) gives two ordinary differential equations

$$\frac{d}{dx}\left\{ y_{2i} + \delta_i \left[m_i A_{1i} + (1 - m_i) A_{2i} \right] \right\} = 0 ,$$
$$\frac{d}{dx}\left[y_{2i} + \delta_i m_i A_{2i} + \delta_i (1 - m_i) A_{3i} \right] = (-1)^{i+1} 2 (1 - m_i) \beta^2 B_i . \tag{1.5}$$

Here

$$A_{1i} = \int_0^1 \frac{\rho}{\rho_0} f(\eta_i)\, d\eta_i , \quad A_{2i} = \int_0^1 \frac{\rho}{\rho_0} f^2 (\eta_i)\, d\eta_i ,$$
$$A_{3i} = \int_0^1 \frac{\rho}{\rho_0} f^3 (\eta_i)\, d\eta_i , \quad B_i = \int_0^1 \frac{\rho}{\rho_0} f^{1,3} (\eta_i)\, d\eta_i \tag{1.6}$$

and it is taken into consideration that

$$(\rho \overline{u'v'})_i = (-1)^{i+1} \rho l^2 \left[\frac{\partial (u - u_{\delta i})}{\partial y} \right]^2 \tag{1.7}$$

(in the upper mixing zone a positive fluctuation v' induces a positive fluctuation u', i.e., $\overline{u'v'} > 0$ and in the lower mixing zone $\overline{u'v'} < 0$, since a negative fluctuation v' induces a positive fluctuation u' and vice versa).

Since the mixing zones are not connected with each other and develop in different conditions, it seems reasonable to assume that the mixing lengths in these zones are different but their ratio to the corresponding width of the mixing zone is constant and the same:

$$\frac{l_i}{\delta_i} = \beta . \tag{1.8}$$

Integration of Eq. (1.5) in x with the conditions

$$y_0 = 0 , \quad y_{2i} = \pm 0.5 , \quad \delta_i = 0 \quad \text{when} \quad x=0$$

and subsequent solution of the obtained algebraic equations relative to δ_i and y_{2i} give

$$\delta_i = (-1)^{i+1} \frac{\varphi (m_i)}{\beta^2} \frac{\beta^2 x}{m_i A_{1i} + (1 - m_i) A_{2i}} ,$$
$$y_{2i} = (-1)^{i+1} \left[0.5 - \frac{\varphi (m_i)}{\beta^2} \beta^2 x \right] , \tag{1.9}$$

where

$$\frac{\varphi (m_i)}{\beta^2} = \frac{2 (1 - m_i) B_i \,[m_i A_{12} + (1 - m_i) A_{2i}]}{m_i (A_{2i} - A_{1i}) + (1 - m_i)(A_{3i} - A_{2i})} . \tag{1.10}$$

With the help of Eq. (1.9) one can find the length of the initial section and the position of the jet axis at the end of the initial section, having required that at the end of the initial section the boundaries of the core in the upper and lower parts of the jet converge at a point, i.e., using the condition

$$y_{21} = y_{22} = y_0 (x_{\text{in}}) \quad \text{at} \quad x = x_{\text{in}} . \tag{1.11}$$

Having written Eqs. (1.9) at $i = 2$, and $x = x_i$, using condition (1.11), and having solved the obtained equations simultaneously relative to x_{in} and $y_0(x_{\text{in}})$, we obtain

$$\beta^2 x_{\text{in}} = \frac{1}{\dfrac{\varphi (m_1)}{\beta^2} + \dfrac{\varphi (m_2)}{\beta^2}} , \quad y_0 (x_{\text{in}}) = \frac{1}{2} \frac{\varphi (m_2) - \varphi (m_1)}{\varphi (m_1) + \varphi (m_2)} . \tag{1.12}$$

The density profile which should be known for calculation of integrals (1.6) will be determined from the equation of state with the assumption of linear temperature variations in the mixing zones [2]:

$$\frac{T - T_{\delta i}}{T_0 - T_{\delta i}} = 1 - \eta_i \tag{1.13}$$

and

$$\frac{\rho}{\rho_0} = \frac{1}{1 - (1 - \theta_i)\,\eta_i}, \tag{1.14}$$

where

$$\theta_i = \frac{T_{\delta i}}{T_0}. \tag{1.15}$$

Integral (1.6) can be taken easily with the help of Eqs. (1.3) and (1.14). The calculated plot of integrals versus temperature ratio (1.15) is shown in Fig. 2. From this plot characteristics of the mixing zones, the length of the starting section, and the position of the axis of the jet at the end of the initial section can be calculated easily from formulas (1.9), (1.10), and (1.12).

As can be seen from Fig. 3, asymmetry of the boundary conditions has a great effect on the characteristics of the jet. It should be noted that the position of the jet at the end of the initial section can change its sign, depending on the temperature ratio at the boundaries of the jet and the length of the initial section can decrease severalfold with a preset m_1.

2. Let a fuel jet flow out of a plane nozzle into an oxidant medium flowing on one side of the jet. It will be assumed that the other side of the fuel jet interacts with combustion products having a low negative velocity that can be assumed zero to a first approximation. As has been mentioned above, this flow pattern occurs in a combustion chamber at the center of which (inside the circular jet) a weak circulating flow of combustion products arises due to ejection of transverse jets in the downstream flow, and a circular flow of oxidant moves between the jet and the wall. In the case of combustion the flame front arises in the external part of the jet (on the side of the oxidant) (see Fig. 1b).

On the flame front the fuel and oxidant concentrations are zero and the transverse flows of the fuel and oxidant are in a stoichiometric ratio

$$L = \frac{\partial \kappa_{\mathrm{ox}}/\partial y}{\partial \kappa_{\mathrm{fu}}/\partial y} \quad \text{at} \quad y = y_{\mathrm{f.f}}. \tag{2.1}$$

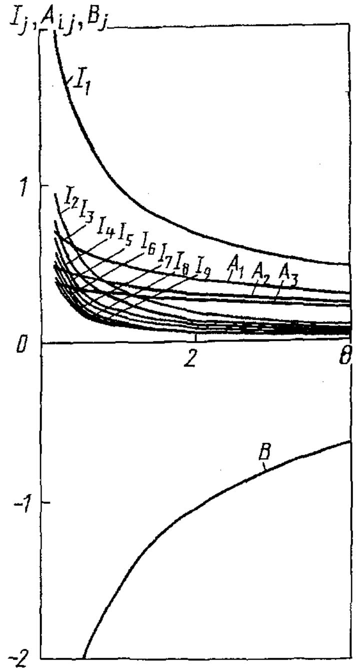
Since a jet with combustion differs from a nonisothermal jet only by the presence of a flame front, all the formulas obtained here earlier for a nonisothermal jet are also valid for a jet with combustion. However, integrals (1.6) should be calculated with combustion taken into consideration since the temperature and density in the mixing zones depend on the density and temperature of the gases on the flame front.

It will be assumed that in the mixing zones the concentrations of the oxidant, combustion products, and fuel change following the linear law from the external boundary to the flame front and from the flame front to the boundary of the potential core in the external mixing zone and from the boundary of the potential core to the boundary of the jet – in the internal mixing zone, i.e.,

$$\kappa_{\mathrm{ox}} = \kappa_{\mathrm{ox}1}\frac{y - y_{\mathrm{f.f}}}{y_{11} - y_{\mathrm{f.f}}}, \quad \kappa_{\mathrm{fu}} = \kappa_{\mathrm{fu}2}\frac{y_{\mathrm{f.f}} - y}{y_{\mathrm{f.f}} - y_{21}}, \quad \kappa_N = \kappa_{N1}\frac{y - y_{21}}{y_{11} - y_{21}}. \tag{2.2}$$

From Eqs. (2.2) and (2.1) we can easily obtain the relation between the stoichiometric ratio L, initial concentrations of the fuel and oxidant, and the ordinates of the boundaries of the upper zone:

$$L = \frac{\kappa_{\mathrm{ox}1}}{\kappa_{\mathrm{fu}1}}\,\frac{y_{\mathrm{f.f}} - y_{21}}{y_{11} - y_{\mathrm{f.f}}}. \tag{2.3}$$



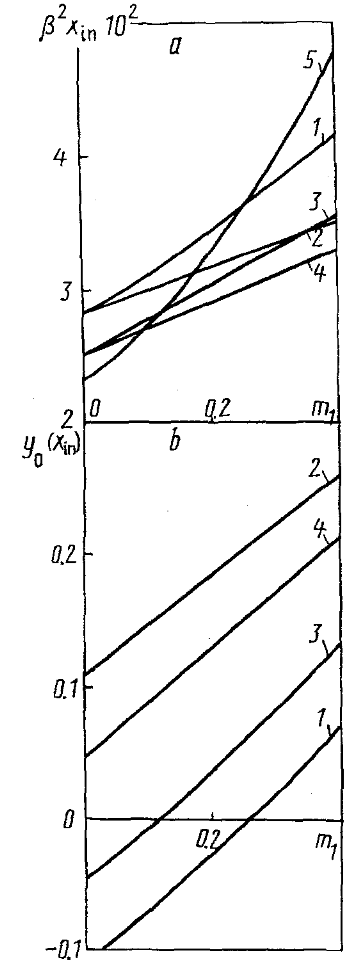


Fig. 2. Plot of the integrals I_j; A_{ij}, and B_j versus the parameter θ.

Fig. 3. Changes in the length of the initial section (a) and the position of the axis of the jet (b) as a function of the parameters m_1, m_2, θ_1, θ_2: at $m_2 = 0$: 1) $\theta = 0.5$; $\theta_2 = 2.0$; 2) $\theta = 2.0$; $\theta_2 = 0.5$; 3) $\theta_1 = 0.5$; $\theta_2 = 1.0$; 4) $\theta_1 = 1.0$; $\theta_2 = 0.5$; at $m_2 = m_1$; 5) $m_1 = m_2 = 0$; 0.2; 0.4; $\theta_1 = \theta_2 = 0.5$.

Using Eq. (2.3), it is possible to determine the ordinate of the flame front at the end of the initial section $y_{\text{f.f.in}}$ assuming $y_{21} = y_0(x_i) = y_{22}$. After simple transformations we obtain

$$y_{\text{f.f.n}} = \frac{L\, y_{11}\, (x_{\text{in}})\, \dfrac{\kappa_{\text{fu}2}}{\kappa_{\text{ox}1}} + y_0\, (x_{\text{in}})}{1 + L\, \dfrac{\kappa_{\text{fu}2}}{\kappa_{\text{ox}1}}}. \tag{2.4}$$

The concentration of the combustion products in the upper mixing zone can be found from the relations following from the definition of the concept of concentration:

$$\begin{aligned} &\kappa_N + \kappa_{\text{ox}} + \kappa_{\text{c.p}} = 1 \quad \text{at} \quad y_{\text{f.f}} < y \le y_{11}\,; \\ &\kappa_N + \kappa_{\text{fu}} + \kappa_{\text{c.p}} = 1 \quad \text{at} \quad y_{21} \le y < y_{\text{f.f}}\,. \end{aligned} \tag{2.5}$$

When determining the concentration of combustion products, it should be borne in mind that the following conditions should be satisfied

$$\begin{aligned} &\kappa_{\text{fu}} = 1\,, \quad \kappa_N = \kappa_{\text{c.p}} = 0 \quad \text{at} \quad y = y_{21}\,; \\ &\kappa_{\text{fu}} = 0\,, \quad \kappa_N = \kappa_{N1}\,, \quad \kappa_{\text{c.p}} = 1 - \kappa_{N1} \quad \text{at} \quad y = y_{\text{f.f}}\,; \end{aligned} \tag{2.6}$$

$$\kappa_N = \kappa_{N1}\,, \quad \kappa_{c.p} = 0\,, \quad \kappa_{ox} = 1 - \kappa_{N1} = \kappa_{ox1} \quad \text{at} \quad y = y_{11}\,.$$

With conditions (2.5) and (2.6) taken into consideration, the concentration of combustion products can be calculated from the formula

$$\kappa_{c.p} = 1 - \kappa_{N1} - \kappa_{ox1}\frac{y - y_{f.f}}{y_{11} - y_{f.f}} \quad \text{at} \quad y_{f.f} < y \le y_{11}\,;$$
$$\kappa_{c.p} = 1 - \kappa_{N1}\frac{y - y_{21}}{y_{11} - y_{21}} - \kappa_{fu2}\frac{y_{f.f} - y}{y_{f.f} - y_{21}} \quad \text{at} \quad y_{21} \le y < y_{f.f}\,. \tag{2.7}$$

In the low combustion zone only combustion products are present, which can include neutral gas and the fuel. Here the following conditions should be satisfied

$$\kappa_{fu} = 0 \quad \text{at} \quad y = y_{12}\,; \quad \kappa_{fu} = \kappa_{fu2} \quad \text{at} \quad y = y_{22}\,. \tag{2.8}$$

Then, with the assumption of linear variation of the fuel concentration, we have

$$\kappa_{fu} = \kappa_{fu2}\frac{y - y_{12}}{y_{22} - y_{12}} \quad \text{and} \quad \kappa_{c.p} = 1 - \kappa_{fu2}\frac{y - y_{12}}{y_{22} - y_{12}}\,. \tag{2.9}$$

It should be noted that the compositions of the combustion products are different in the upper and lower zones, which should be taken into consideration in determination of the density of gases in each of the mixing zones (this density should be known for calculation of the integrals).

The ratio of the densities of the gas mixture in the cross-sections of the mixing zone can be found from the formula (see [1]) obtained with the equation of state for a gas mixture:

$$\frac{\rho}{\rho_0} = \frac{T_0}{T} = \frac{1}{\sum\limits_i \left(\dfrac{\kappa_i}{\kappa_{fu2}}\right)\left(\dfrac{\mu_{fu2}}{\mu_i}\right)}\,. \tag{2.10}$$

Here κ_i and μ_i are the concentration and specific weight of each of the components of the mixture; ρ_0 and T_0 are the density and temperature of the fuel in the core of the jet.

The temperature profile in the cross-sections of the initial section of the jet is assumed to be linear in the cross-section of the jet from the flame front to the external boundary of the jet and from the flame front to the boundary of the core of the jet:

$$\frac{T - T_{fu2}}{T_{f.f} - T_{fu2}} = \frac{y - y_{21}}{y_{f.f} - y_{21}} \quad \text{at} \quad y_{21} \le y < y_{f.f}\,;$$
$$\frac{T - T_{ox1}}{T_{f.f} - T_{ox1}} = \frac{y_{11} - y}{y_{11} - y_{f.f}} \quad \text{at} \quad y_{f.f} < y \le y_{11}\,. \tag{2.11}$$

The temperature of gases on the flame front can be determined from the balance of enthalpies (see [1]). However, it is known experimentally that in combustion of hydrocarbon fuels and hydrogen this temperature changes in a very narrow range and it can be assumed to be known and equal to 2000 K.

Since in the mixing zone from the boundary of the jet to the flame front and from the flame front to the core of the jet, the gas composition is different, it is evident that integrals (1.6) should be divided into two integrals to be calculated separately.

Table 1 contains the calculated lengths of the initial section, rates of expansion of the external and internal mixing zones δ_1' and δ_2', shift of the jet axis at the end of the initial section $y_0(x_{in})$, and the boundaries of the jet for a submerged isothermal jet $m_1 = m_2 = 0$, an isothermal jet with $m_1 = 0.2$ and $m_2 = 0$, a nonisothermal jet with $m_1 = 0.2$ and $m_2 = 0$, $T_{\delta 1}/T_0 = 6.67$, and a jet of methane in an air flow at $T_{f.f}/T_{fu} = 6.67$; $m_1 = 0.2$, and $m_2 = 0$. It is assumed here that the temperature of combustion products on the boundary of the internal mixing zone is equal to the temperature on the flame front. For a jet with combustion, the ordinate of the flame front determined from formula (2.4) is given in Table 1. It can be seen that under similar conditions the length of the initial section

TABLE 1. Calculated Characteristics of the Flow

Jet	x_{in}	δ_1'	δ_2'	$y_0(x_{in})$	y_{11}	y_{12}	$y_{f.f}$
Isothermal jet, $m_1 = m_2 = 0$	5.09	0.264	0.264	0	1.34	1.34	–
Isothermal jet, $m_1 = 0.2$, $m_2 = 0$	5.98	0.173	0.264	0.09	1.12	1.49	–
Nonisothermal jet, $m_1 = 0.2$, $m_2 = 0$, $T_{\delta_1}/T_0 = T_{\delta_2}/T_0 =$ 6.67	14.34	0.147	0.205	0.07	2.18	2.86	–
Jet with combustion $T_{f.f}/T_0 = T_{c.p}/T_0 =$ 6.67, $m_1 = 0.2$, $m_2 = 0$	12.69	0.15	0.216	0.08	1.98	2.66	1.94

of the nonisothermal jet and the jet with combustion is more than twice as long as the length of the initial section of the isothermal submerged jet and the isothermal jet in asymmetric conditions. The difference between the characteristics of the initial section of the jet with combustion and the nonisothermal jet in a flow with a temperature equal to the temperature on the flame front is slight. It can be also seen that the ordinate of the flame front is close to the ordinate of the boundary of the jet and therefore in calculation of the integrals in the equations, it can be assumed for simplicity that the ordinates of the flame front and the boundary of the jet coincide in the initial section.

In conclusion it should be noted that the present method can be used to calculate in a very simple way the characteristics of a complicated flow occurring in the initial section of a plane jet developing in asymmetric conditions. The effect of asymmetry of the boundary conditions is so great that it should be included obligatorily under practical conditions.

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NOTATION

b_0, width of the slot; L, stoichiometric ratio; m, ratio of the flow velocity to the velocity in the jet core; R_c, radius of the chamber; T, temperature; u, velocity; x, y, instantaneous longitudinal and transverse coordinates; x_{in}, length of the initial section; y_0, ordinate of the jet at the end of the initial section; $y_{f.f}$, ordinate of the flame front; $\beta = l_i/\delta_i$; δ, width of the mixing zone; κ, concentration; μ, specific weight of the gas; ρ, density of the gas. Subscripts: 0, core of the jet; 1, 2, inside and outside of the jet; 11, 12, refers to internal and external boundaries of the mixing zone from outside of the jet; 12, 11, refers to the internal and external boundaries of the mixing zone from inside of the jet; $i = 1$; N, neutral gas; fu, fuel; in, initial section; ox, oxidizer; c.p, combustion products; f.f, flame front; $\delta 1$, $\delta 2$, refer to a parameter on the internal and external boundaries of the jet.

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